Numerical Computing
Spring 2007, Homework 6
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The following code implements the secant method.

```
g1 = input('Initial Value 1: ');
g2 = input('Initial Value 2: ');
x = [g1 g2];
b = froot(x(1));
k = 2;
while (abs(x(k)-x(k-1)) > .0001)
    a = b;
    b = froot(x(k));
    if (b-a == 0)
        break;
    end;
    x(k+1) = x(k)-(b*(x(k)-x(k-1)))/(b-a);
    k = k+1;
end;
x
```

Using it, the roots of the cubic equation,

$$f(x) = x^3 + 11x^2 + 26x + 16 = 0$$

are found to be -8, -2, and -1.

The following chart depicts the roots accessed by initial values i (rows) and j (cols). (Where $i = j$, the iteration encounters division by zero.)

```
   |-12 -11 -10  -9  -8  -7  -6  -5  -4  -3  -2  -1   0   1   2   3   4
---|-----------------------------------------------------------------------
-12|     -8  -8  -8 :-8 :-8  -8  -1  -2  -2 :-2 :-1 :-1  -1  -1  -2  -2
-11| -8      -8  -8 :-8 :-8  -8  -1  -2  -2 :-2 :-1 :-1  -1  -2  -2  -1
-10| -8  -8      -8 :-8 :-8  -8  -8  -2  -2 :-2 :-1 :-1  -2  -2  -8  -8
 -9| -8  -8  -8     :-8 :-8  -8  -8  -2  -2 :-2 :-1 :-2  -8  -8  -8  -8
...|................:...:...................:...:...:..................
 -8| -8  -8  -8  -8 :   :-8  -8  -8  -8  -8 :-2 :-1 :-8  -8  -8  -8  -8
...|................:...:...................:...:...:..................
 -7| -8  -8  -8  -8 :-8 :    -8  -1  -2  -1 :-2 :-1 :-1  -1  -2  -8  -8
 -6| -8  -8  -8  -8 :-8 :-8      -8  -1  -2 :-2 :-1 :-1  -1  -2  -8  -8
 -5| -8  -8  -8  -8 :-8 :-8  -8      -2  -2 :-2 :-1 :-1  -1  -8  -8  -8
 -4| -2  -1  -1  -8 :-8 :-8  -1  -2      -2 :-2 :-1 :-1  -8  -8  -8  -1
 -3| -2  -2  -2  -2 :-8 :-1  -2  -2  -2     :-2 :-1 :-8  -8  -2  -2  -2
...|................:...:...................:...:...:..................
```

```
 -2| -2  -2  -2  -2 :-8 :-2  -2  -2  -2  -2 :   :-1 :-2  -2  -2  -2  -2
...|................:...:...................:...:...:...................
 -1| -1  -1  -1  -1 :-8 :-1  -1  -1  -1  -1 :-2 :   :-1  -1  -1  -1  -1
...|................:...:...................:...:...:...................
  0| -1  -1  -1  -2 :-8 :-8  -8  -8  -2  -8 :-2 :-1 :    -1  -1  -1  -1
  1| -1  -1  -2  -2 :-8 :-8  -8  -1  -8  -2 :-2 :-1 :-1      -1  -1  -1
  2| -1  -2  -2  -8 :-8 :-8  -8  -8  -2  -2 :-2 :-1 :-1  -1      -1  -1
  3| -2  -2  -1  -8 :-8 :-8  -8  -8  -2  -2 :-2 :-1 :-1  -1  -1      -1
  4| -2  -1  -8  -8 :-8 :-8  -8  -8  -2  -2 :-2 :-1 :-1  -1  -1  -1
```

As an exercise, we wish to characterize the behavior of the fixed-point iterative scheme for the above function, rewritten in the form,

$$g(x) = -11 - \frac{26}{x} - \frac{16}{x^2}$$

It is known that for a solution x^*, if $x^* = g(x^*)$ and $|g'(x^*)| < 1$, then the iterative scheme is locally convergent. However if $|g'(x^*)| > 1$, then the iterative scheme is divergent for all starting points other than x^*. Noting

$$g'(x) = \frac{26}{x^2} + \frac{32}{x^3}$$

and having computed the roots to be -8, -2, and -1, we know that since $|g'(-8)| = \frac{11}{32} < 1$, the iteration is able to converge toward the root, -8. However, since $|g'(-2)| = \frac{5}{2} > 1$ and $|g'(-1)| = 6 > 1$, the iteration cannot converge toward the other roots, -2 and -1.

To exemplify, we (successfully) compute the root -8 using the fixed-point iterative scheme, starting at the initial value -24:

$$\begin{aligned}
x_1 &= -9.94 \\
x_2 &= -8.55 \\
x_3 &= -8.18 \\
x_4 &= -8.06 \\
x_5 &= -8.02 \\
x_6 &= -8.01 \\
x_7 &= -8.00
\end{aligned}$$

Meanwhile, we attempt but fail to compute the other roots, even when starting at a relatively close initial value -1.5. Instead, the iteration converges toward the root, -8:

$$x_1 = -0.78$$
$$x_2 = -4.02$$
$$x_3 = -5.52$$
$$x_4 = -6.82$$
$$x_5 = -7.53$$
$$x_6 = -7.83$$
$$x_7 = -7.94$$

Finally, as a separate exercise, we consider the roots of

$$f(x) = \frac{1}{\log x} - x \sin x \qquad x \in \mathbb{R}^+ \cup 0$$

With a straightforward modification to our program, we find one root to be 1.7770 using initial values 1.5 and 2, and another root to be 2.7828 using initial values 2 and 3. Plotting the terms of $f(x)$ helps verify our results: