Numerical Computing
Spring 2007, Homework 8
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We wish to construct a collocation fit using Chebyshev polynomials for

$$f(t) = (t+1)e^{-2t^2} \qquad -1 < t < 1$$

using the Chebyshev points

$$t_3 : 0, \pm\frac{1}{\sqrt{2}}$$
$$t_4 : \pm\frac{1}{\sqrt{2}}, \pm\frac{3}{\sqrt{2}}$$
$$t_5 : 0, \text{roots of } 8t^4 - 8t^2 + 1 = 0$$
$$t_6 : \pm\frac{3}{\sqrt{2}}, \text{roots of } 16t^4 - 20t^2 + 5 = 0$$

In order to do so, we require several component functions.

The first function returns, for a vector t, the vector $[f(t_1)\ f(t_2)\ \cdots\ f(t_n)]$:

```
function [res] = hw8_f(t)
res = [];
for i = 1:1:size(t)
    res = [res; (t(i)+1)*exp(-2*t(i)^2)];
end;
```

The second function returns the nth Chebyshev polynomial, $0 \leq n \leq 5$:

```
function [res] = hw8_T(n,t)
if n==0
    res = 1;
elseif n==1
    res = t;
elseif n==2
    res = 2*t^2-1;
elseif n==3
    res = 4*t^3-3*t;
elseif n==4
    res = 8*t^4-8*t^2+1;
elseif n==5
    res = 16*t^5-20*t^3+5*t;
else
    res = 0;
end;
```

The third function returns the nth set of Chebyshev points, $3 \leq n \leq 6$, normalized to fit the interval $[1, 1]$:

```
function [res] = hw8_tk(N)
if N==3
    res = [-1/sqrt(2) 0 1/sqrt(2)]';
elseif N==4
    res = [-3/sqrt(2) -1/sqrt(2) 1/sqrt(2) 3/sqrt(2)]';
elseif N==5
    p = [8 0 -8 0 1]';
    res = sort([0; roots(p)]);
elseif N==6
    p = [16 0 -20 0 5];
    res = sort([-3/sqrt(2); 3/sqrt(2); roots(p)]);
else
    res = [];
end;
res = res/norm(res,inf);
```

The last function, though less separable, is factored out for convenience:

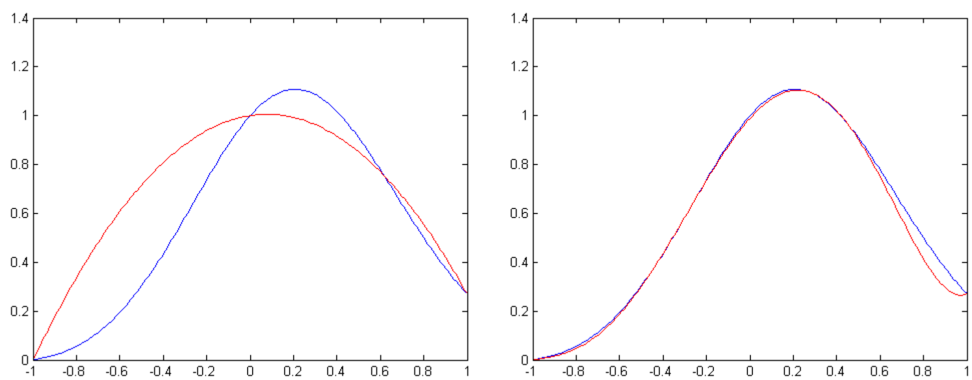
```
function [res] = hw8_p(N,x,t)
res = 0;
for i = 0:1:N-1
    res = res + x(i+1)*hw8_T(i,t);
end;
```

Finally, we may solve the problem:

```
clear;
N = 0;
while ~(N==3 || N==4 || N==5 || N==6)
    N = input('N? (3..6) ');
end;
t = hw8_tk(N);
A = [];
b = [];
for j=0:1:N-1
    tmp = [];
    for i=0:1:N-1
        tmp = [tmp hw8_T(i,t(j+1))];
    end;
    A = [A; tmp];
    b = [b; hw8_f(t(j+1))];
end;
x = A\b;
m = [];
n = [];
```

```
for i=-1:.01:1
    m = [m; i];
    n = [n; hw8_f(i)];
end;
plot(m,n);
hold on;
m = [];
n = [];
for i=-1:.01:1
    m = [m; i];
    n = [n; hw8_p(N,x,i)];
end;
plot(m,n,'r');
hold off;
```

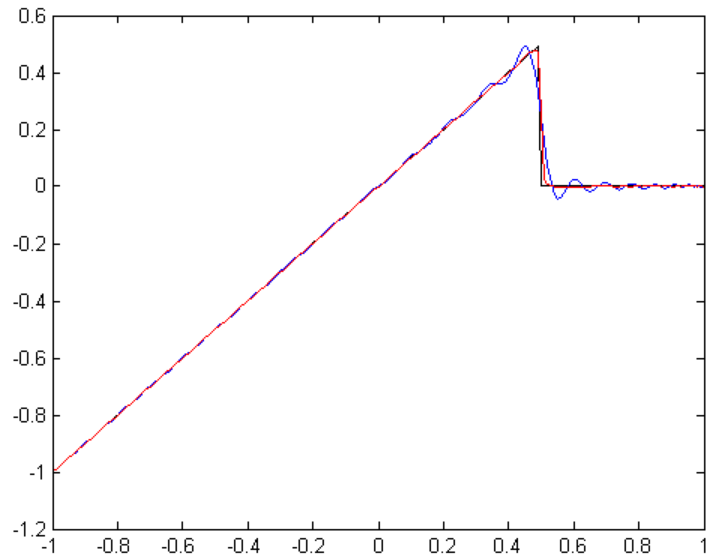
Below, we plot the true function (blue) against the interpolating polynomial (red) for cases $N = 3$ (left) and $N = 6$ (right).



For further exercise, we construct the Chebyshev series for another function,

$$f(t) = \begin{cases} t & : -1 < t < \frac{1}{2} \\ 0 & : \frac{1}{2} < t < 1 \end{cases}$$

and plot the original function, a 50-term approximation, and a 500-term approximation, on one graph. The plot follows:


0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1