Fundamentals of Mathematics
Spring 2007, Proof 6
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Let x be a non-zero element of the field $\mathcal{F}$. Prove that x has at most one multiplicative inverse.

Proof. Let $x \in \mathcal{F}$ and let x^{-1} and x_*^{-1} denote elements of $\mathcal{F}$ such that $x \cdot x^{-1} = 1 = x \cdot x_*^{-1}$. Using the axioms of multiplicative identity, inverse, and associativity, we argue that $x^{-1} = x_*^{-1}$ with the following computation:

$$\begin{aligned} x^{-1} &= x^{-1} \cdot 1 \\ &= x^{-1} \cdot (x \cdot x_*^{-1}) \\ &= (x^{-1} \cdot x) \cdot x_*^{-1} \\ &= 1 \cdot x_*^{-1} \\ &= x_*^{-1} \end{aligned}$$

This completes the proof.